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Number of Terms.	Mid-Term or Pair of Mid-Terms.	The Series.
1	15	15
3	5	4+5+6
5	3	1+2+3+4+5
15	1	-6-5-4-3-2-1+0+1+2+3+4+5+6+7+8
2	7,8	7+8
6	2,3	0+1+2+3+4+5
10	1,2	-3-2-1+0+1+2+3+4+5+6
30	0,1	-14-13- . . . +0+1 . . . +14+15

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## NOTE ON PRIME NUMBERS.

By DERRICK N. LEHMER, University of California.

It is a well known theorem that it is possible to find an arbitrarily great number of consecutive composite numbers. This appears from the values which the expression  $n!+r$  takes for  $r=2, 3, \dots, n$ . This theorem furnishes an interesting proof of the theorem that the number of primes less than or equal to  $x$  is not determined by a function of  $x$  which is a polynomial in  $x$  of finite degree. For if  $f(x)$  were such a function of degree  $n$ , then for  $x=(n+2)!+r$ ,  $f(x)$  must keep the same value for  $r=2, 3, 4, \dots, n+2$ . If this value is  $k$ , then  $f(x)-k=0$  is an equation of degree  $n$  with  $n+1$  roots, which is impossible.

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## DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

### ALGEBRA.

362. Proposed by JAMES F. LAWRENCE, Stillwater, Okla.

Show that the number of solutions in positive integers, zero included, of the equation  $x+2y+3z=6n$ , is  $3n^2+3n+1$ .

Solution by W. J. GREENSTREET, M. A., Editor of the Mathematical Gazette, Burghfield, England.

$x+2y+3z=6n$ .  $z$  may have any value from 0 to  $2n$ , inclusive.

Hence we may assign to it any even or odd value from 0 to  $2n$ , inclusive.

i. Let  $z=2r$ .  $x+2y=6(n-r)$ .  $y$  may have any value from 0 to  $3(n-r)$ .

$\therefore$  There are  $3(n-r)+1$  solutions when  $z$  is  $2r$ .

$\therefore$  Total number of solutions for  $z$  even is

$$\begin{aligned} \sum_{r=0}^{r=n} [3(n-r)+1] &= (3n+1)(n+1) - \frac{3n(n+1)}{2} \\ &= \frac{n+1}{2}(6n+2-3n) = \frac{(n+1)(3n+2)}{2}. \end{aligned}$$

ii. Let  $z=2r+1$ .  $x+2y=6(n-r)-3$ .  $y$  may have any value from 0 to  $3(n-r)-2$ .

$\therefore 3(n-r)-2+1=3(n-r)-1$  solutions.

$\therefore$  Total number of solutions when  $z$  is odd:

$$\sum_{r=0}^{r=n-1} [3(n-r)-1] = (3n-1)n - \frac{3n(n-1)}{2} = \frac{n}{2}(6n-2-3n+3) = \frac{n}{2}(3n+1).$$

$\therefore$  The total number of solutions

$$= \frac{(n+1)(3n+2)}{2} + \frac{n(3n+1)}{2} = \frac{3n^2+5n+2+3n^2+n}{2} = 3n^2+3n+1.$$

Also solved by H. Prime, J. Scheffer, H. C. Feemster, and A. M. Harding.

363. Proposed by E. B. ESCOTT, Ann Arbor, Mich.

(a) If  $a$  and  $n$  be positive integers, the integral part of  $[a+\sqrt{a^2-1}]^n$  is odd.

(b) If  $a$  and  $n$  be positive integers, the integral part of  $[\sqrt{a^2+1}+a]^n$  is odd when  $n$  is even and even when  $n$  is odd. [From Todhunter's *Algebra*, p. 353].

#### I. Solution by the PROPOSER.

*Proof.* (a) Let  $[a+\sqrt{a^2-1}]^n = P + Q\sqrt{a^2-1} = m$ .

Then  $[a-\sqrt{a^2-1}]^n = P - Q\sqrt{a^2-1} = 1/[a+\sqrt{a^2-1}]^n$ .

$\therefore 0 < P - Q\sqrt{a^2-1} < 1$ .

Adding  $m$  to each member of the inequality  $m < 2P < m+1$ .

Therefore, the integral part of  $m$  is odd.

(b) Let  $[\sqrt{a^2+1}+a]^n = R + S\sqrt{a^2+1} = k$ .

Then  $[-\sqrt{a^2+1}+a]^n = R - S\sqrt{a^2+1} = \left(\frac{-1}{\sqrt{a^2+1}+a}\right)^n$ .

If  $n$  is even,  $0 < R - S\sqrt{a^2+1} < 1$ . Adding  $k$ ,  $k < 2R < k+1$ . Whence the integral part of  $k$  is odd.